ELSEVIER
Discrete Mathematics 201 (1999) 1–3
DISCRETE MATHEMATICS

Editorial

In January of 1997, the American Mathematical Society and the Mathematical Association of America held their annual joint meeting in San Diego, California, and I had the privilege of presenting a plenary lecture on Partially Ordered Sets. My good friend and colleague, Hal Kierstead, helped me to organize a minisymposium on the same theme. Later that year, Peter Hammer invited me to serve as Guest Editor for a special issue of Discrete Mathematics on Partially Ordered Sets as a follow-up to the San Diego activities — and I was delighted to accept his invitation. This volume is the end result.

Partially ordered sets have long been one of my main research interests — starting from the NSF sponsored 1971 combinatorics conference held at Bowdoin College. And it is particularly pleasing to see how research on these fascinating structures has evolved and deepened over the years. I would never claim that one lecture, one minisymposium and one volume can be exhaustive in their coverage, but this collection of papers provides a good cross-section of research trends and directions. It will also provide students and researchers with an overview of modern research techniques and concepts for posets.

A few words about the papers in this volume. There are two invited survey papers. First, Graham Brightwell has done us all a good turn by providing a unified treatment of the many research results related to the concepts of balanced pairs and sorting with partial information — a subject to which he has also been one of the main contributors. The original motivating problem here is the still unsolved 1/3–2/3 conjecture of Kislitsyn: In any poset P, there is always a pair of points x, y for which $1/3 \leq \text{Prob}[x<y] \leq 2/3$. Brightwell's article includes a concise treatment of the original breakthrough paper by Kahn and Saks, as well as the more recent refinements which yield the best possible result among the countably infinite posets. Brightwell also surveys the extensive application of geometric techniques to balancing pairs, and gives additional details on the connections with sorting.

Second, Hal Kierstead has given a compact treatment of the problem of determining the dimension of two levels of the subset lattice. It is truly surprising how many applications there are for this line of research. Connections with packing and covering problems are immediate, but recent work has established connections with topics which at first glance seem far afield. This article will appeal to students and young researchers in particular.

On the computer science front, Russ Bubley and Martin Dyer have produced a very important result on generating random linear extensions, a problem with many

PII: S0012-365X(98)00308-2

applications. They describe a new Markov chain for generating extensions and use path coupling to prove that it is rapidly mixing. The mixing time is $O(n^3 \log n)$, a significant improvement over earlier bounds.

Ross McConnell and Jeremy Spinrad provide the best known bounds for modular decomposition and transitive orientation of a graph. The task of providing a modular decomposition of a graph occurs in many settings, as does the challenge of ordering the edges transitively. As corollaries, they obtain linear time bounds for recognizing permutation graphs and comparability graphs.

Dwight Duffus and Bill Sands investigate the sizes of prime filters in distributive lattices. They also show that finite distributive non-Boolean lattices always contain a prime filter which consists of at least one-third and at most two-thirds of the points in the lattice. This work is motivated at least in part by efforts to solve the famous Frankl conjecture: every union closed family has a point in at least half the sets.

Serkan Hoşten and Walter Morris have made a a particularly elegant contribution with their analysis of the dimension of the complete graph. This is equivalent to determining the dimension of the poset formed by the 1-element and 2-element subsets of a set. The clean translation into an extremal problem closely related to Dedekind's problem is quite appealing and the fact that it permits one to calculate this parameter for $n \leq 229809982113$ is truly amazing.

Brightwell, David Grable and Hans Jürgen Prömel investigate an important new direction involving asymptotic enumeration of posets which exclude a forbidden poset. They show that these enumeration problems can be naturally grouped into four classes, and they show that if one forbids any poset which is an antichain or one of 10 small posets, then the number of labelled posets on n points is at most $n!c^n$. Some intriguing open problems remain.

Ken Bogart and Doug West show that elegance is also a feature of arguments for posets with their short proof that interval graphs with proper representations are just the same as those which admit a representation where all intervals have the same length.

Zbigniew Lonc provides some new insights on the challenging extremal problem of finding the smallest fibre in a poset, a set of points which intersects each non-trivial maximal antichain. Lonc finds some common properties satisfied by width 3 posets for which the hypergraph of non-trivial maximal antichains is not 2-colorable.

Utz Leimich and Klaus Reuter investigate fractional dimension — the linear programming relaxation of the integer valued paramter. They give a nice formula for the fractional dimension of two levels of the Boolean lattice, show that it is invariant under completions, and analyze the product of standard examples.

I am a co-author on two articles in this volume — something I did not intend to do when I set out on this project. But with Stefan Felsner and Peter Fishburn, we solved one of the long standing open problems involving posets by showing that there exists a finite three dimensional poset which is not a sphere order. This result settles two conjectures. First, it shows that not every finite three dimensional poset is a circle order, and second, it shows that not every finite poset is a sphere order. The argument is complex, but the Ramsey theoretic reasoning should have further applications.

In joint work with Geir Agnarsson and Stefan Felsner, we investigated a natural extremal problem for graphs, finding the maximum number of edges in a graph of given dimension. Combinatorists will enjoy the application of such classical techniques as the Erdős/Stone theorem and the product Ramsey theorem in this paper, while algebraists will appreciate the ring theoretic applications which motivated Agnarsson to propose this line of research in the first place.

So, what is not here. First, this volume does not cover traditional lattice theory. Second, it does not contain papers on Sperner theory or extremal set theory. Third, there are no papers on the important connections with algebra, Cohen/Macaulay posets and the like. All these areas enjoy a rich history with vibrant, active research underway today.

In reading the papers in this volume, I hope that researchers will gain an appreciation of the variety, scope and significance of modern research efforts for partially ordered sets. It is certainly the case that good work is being done, but it is also the case that the best is still to come.

Please allow me to thank each of the contributors to this volume for their efforts. I would also like to thank Arizona State University, the Freie Universität Berlin, the Office of Naval Research and the Deutsche Forschungsgemeinschaft for their support during my sabbatical year here in Berlin.

William T. Trotter
Guest Editor
Department of Mathematics
Arizona State University
Temple, AZ 85287
USA
E-mail: trotter@asu.edu